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MATHEMATICAL PROPERTIES OF A TUBERCULOSIS MODEL WITH TWO DIFFERENTIAL INFECTIVITY AND N LATENT CLASSES

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Abstract. This paper considers the global properties of a very general tuberculosis model with two differential infectivity, n classes of latent individuals and mass action incidence. This general system exhibits the traditional threshold behavior. There is always a globally asymptotically stable equilibrium state. Depending on the value of the basic reproduction ratio $\mathcal{R}_0$, this state can be either endemic ($\mathcal{R}_0 > 1$), or infection-free ($\mathcal{R}_0 \leq 1$). The global stability of this model is derived through the use of Lyapunov stability theory and LaSalle's invariant set theorem. Computer simulations are given to illustrate analytical results.

Keywords. Nonlinear dynamical systems, Epidemiological models, Tuberculosis models, Global stability, Lyapunov functions.

1 Introduction

Tuberculosis (TB) is primarily a disease of the respiratory system with variable degrees of infectiousness. It can follow infection with the airborne bacteria germ *Mycobacterium tuberculosis*. Bacilli only live in the air for approximately 2 hours so individuals who have intense contact with TB bacilli in poorly ventilated areas are the most likely to become infected. Thus, TB morbidity and mortality rates are strongly affected by living conditions. Infectiousness of the source case, duration and frequently of exposure and characteristics of shared environments, all contribute to the overall risk of transmission [1-10]. It is also known that factors such as endogenous reactivation, emergence of multi-drug resistant TB, and increase in HIV incidence in the recent years call for improved control strategies for TB. Another issue that is essential to the epidemiology of TB is the exogenous re-infection, where a latently-infected individual acquires a new infection from another infectious (see [11-13] and references therein).

Many mathematical models for tuberculosis differentiate between individuals according to their history of infection. In particular, the population under investigation is subdivided into four epidemiological classes: susceptible individuals, latently infected individuals (those who are infected but not infectious), infectious and the recovered or cured individuals [3,17]. It should be pointed out that according to the Direct Observation Therapy Strategy (DOTS) applied in most developed countries, a patient with pulmonary tuberculosis must make three sputum examinations during his treatment and will be considered cured when the last result of the examination of sputum is negative. Some researchers, however, model tuberculosis taking into consideration three of the four epidemiological classes (excluding the recovered class) [1,9,18], with the assumption that recovered individuals revert back to the latent class. These models, irrespective of the number of epidemiological classes, have given rise to interesting results both qualitatively and quantitatively. However, the division of a population into various compartments give rise to compartmental models (whose complexity increases with increasing compartments, interactions compartments, interactions and specific population characteristics such as age, periodicity, susceptibility, infectivity, etc..).

On the other hand, it is well known that the duration of latency varies greatly from case to case [14]. It is possible for a tuberculosis infection to become active within a few months of infection. It is also possible that activation may occur several years or decades after exposure has taken place. Until such time, the individual suffers no ill-effects of the disease, and cannot transmit the disease to others [14,15]. Also, the risk of activation seems to decrease over time [11,12]. One way in which this can be modeled is to include a sequence of several latent classes through which latently infected individuals can pass. Each latent class can be assigned its own activation rate. TB infected individuals, generally classified as "infective", play a major

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