Journal of Nonlinear Systems and Applications (2011) 111–121

http://www.watam.org/JNSA/

INTERCONNECTIONS OF HYBRID SYSTEMS: SOME CHALLENGES AND RECENT RESULTS

Ricardo G. Sanfelice *†

Abstract. This paper provides an overview of the problem of interconnecting hybrid systems. Hybrid systems are given in terms of constrained differential and difference equations with inputs and outputs. Issues on existence of solutions and mismatch of time domains of their solutions are discussed. An input/output stability notion for such systems is presented and a small gain theorem for analyzing interconnections of hybrid systems is presented.

Keywords. hybrid systems, interconnections, input/output notions, stability.

1 Introduction

A supervisory algorithm selecting the most appropriate control law for the current operating conditions of a plant, an event-driven transmission of information between networked agents, and an (almost) instantaneous change of velocities in mechanical systems with impacts are just a few representative systems that can be studied using hybrid systems theory. The prevalent combination of continuous and discrete dynamics in systems in science and engineering makes hybrid systems a powerful modeling framework and setting for analysis, design, and simulation.

The understanding of the dynamical properties of hybrid systems has been possible using machinery inspired from control theory, as it has been demonstrated in [45, 27, 13]. Particular attention has been paid to the analysis of *closed hybrid systems*, that is, those without inputs. Tools for the analysis of such systems have permitted the study of Lyapunov stability, convergence, and robustness properties. These are particularly useful for the analysis of the closed-loop systems resulting from feedback control, in which the required properties of the control algorithm are inferred from the desired properties of the closed-loop system. *Open hybrid systems* are systems with inputs, such as disturbances and control inputs, and outputs. Recent results on input-to-state stability for hybrid systems [7] permit the study of bounds on the state trajectories in terms of the initial conditions and inputs, similarly to their counterparts for continuous-time systems [40], discrete-time systems [21], and switched systems [28, 44].

Tools for closed systems are applicable for the analysis of interconnected systems, but typically do not scale with the size of the interconnection. The understanding of the properties of open systems are particularly useful in the study of the properties of their interconnections. For instance, certain properties conferred by a controller to a closed-loop control system can be determined from the properties of the individual (interconnected) plant and controller. Recent results reported in [31] and [10] show that the small gain theorem in [20] for continuous-time systems can be formulated in the hybrid setting to assert that interconnections of input-to-state stable hybrid systems are input-to-state stable. This motivates the study of notions relating hybrid inputs and outputs, such as input-output boundedness [11, 22] and input-to-output stability [41, 37], as well as the generation of tools for the analysis of input/output properties of interconnections of hybrid systems.

The purpose of this paper is to provide an overview of the problem of interconnecting hybrid systems and to present some recent results for input/output analysis. More precisely, given hybrid systems $\mathcal{H}_1$ and $\mathcal{H}_2$ with certain stability properties in an input/output (I/O) sense, we explore tools for the analysis of interconnections of hybrid systems, such as those in Figure 1. To this end, we introduce, in a tutorial tone, a framework for hybrid systems in Section 2 and key issues for the study of their interconnections in Section 3. Then, in Sections 4 and 5, we present results on input-to-output stability and a small gain theorem for the analysis of input-to-output stable interconnections. Examples throughout the paper illustrate the ideas.

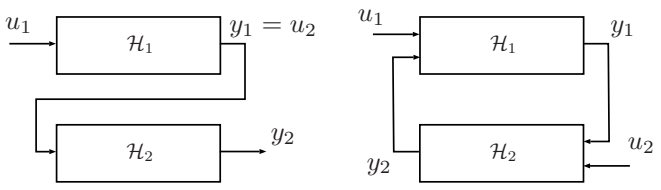

Figure 1: Series and feedback interconnections of two hybrid systems.

*R. G. Sanfelice is with the Department of Aerospace and Mechanical Engineering, University of Arizona, 1130 N. Mountain Ave, AZ 85721. Email: sricardo@u.arizona.edu

†Manuscript received February 17, 2011; revised April 8, 2011.