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OPERATOR VALUED MEASURES AS FEEDBACK CONTROLS FOR STOCHASTIC SYSTEMS ON HILBERT SPACE

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Abstract. In this paper we present a summary of the talk presented at the workshop which is based on the results from a recent paper of the author. We consider a linear stochastic system on infinite dimensional Hilbert space with structural feedback control based on partial observation corrupted by an arbitrary second order centered random process. The admissible feedback controls consists of a weakly compact set from the space of operator valued measure. We prove existence of optimal controls.

Keywords. Partially Observed, Stochastic Control, Hilbert space, Operator Valued Measures, Hybrid System.

1 Introduction

Other than pure mathematical motivation [2,3,4], there are important physical motivations arising from different applications. For example, polymerization requires control of molecular structure. This is done by use of laser beam shot into the materials to reorganize the molecular structure of the system. The system dynamics is governed by Schrödinger Equation with controlled potential. Another example is provided by stochastic structural mechanics such as communication Satellites with Flexible Appendages subject to bombardments by micro meteorites. Interesting application is found in fabrication of meta materials for optical filters where Maxwell Equation subject to disturbance provides the dynamics. There are other areas such as aerospace engineering where structural controls are used routinely to maneuver aircrafts. Abrupt phase changes, such as transition from liquid to solid state or solid to liquid state, in response to magnetic or electric fields, as exhibited by magneto-rheostatic or electro-rheostatic materials, are good examples of physical sciences where structural controls are natural.

The rest of the paper is organized as follows. In section 2, we present the problem considered in this paper. Some notations and definitions are given in section 3. In section 4, the system dynamics with feedback control operator is described. In section 5, the question of existence of evolution operators is considered. Section 6, is devoted to the questions of continuity and regularity properties of solutions. Existence of optimal (structural) controls is given in section 7. Detailed proof of all the results presented here can be found in the recent paper of the author [1].

2 Problem statement and preliminaries

In this paper we are interested in partially observed stochastic control problem with structural controls which are operator valued measures[1]. This is described by a SDE on Hilbert space coupled with an algebraic equation representing noisy measurement process as follows:

$$dx = Axdt + B(dt)y(t-) + \sigma(t)dW(t), x(0) = x_0, \quad (1)$$

$$y(t) = C(t)x(t) + \xi(t), t \in I = [0, T]. \quad (2)$$

Control Problem: We are interested in (a): control of fluctuation, (b): control of the mean trajectory, and (c): control of structural variation. All these concerns are covered by the following objective functional,

$$J(B) \equiv E \int_I |x(t) - \bar{x}(t)|_X^2 \lambda(dt) + \int_I |\bar{x}(t) - x_d(t)|_X^2 \nu(dt) + \Phi(B), \quad (3)$$

where $\bar{x}(t)$ denotes the mean trajectory and x_d is the desired trajectory. The functional Φ is a measure of cost associated with the total variation of the measure B.

3 Some Notations

Let $I \equiv [0, T]$ be a finite interval and Σ the sigma algebra of subsets of the set I. We use the following spaces of operator valued measures:

$$M_{ca}(\Sigma, \mathcal{L}(Y, X)) \supset M_{cabv}(\Sigma, \mathcal{L}(Y, X)) \supset M_{rcabv}(\Sigma, \mathcal{L}(Y, X)),$$

where the first one is the space of countably additive (in the strong operator topology: τ_{so}) operator valued measures, the second is the same having bounded variations,

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[1]The results of this manuscript was presented at the Workshop on Hybrid Dynamic Systems, Waterloo University, July 29-31, 2010.