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VIRUSLIKE DYNAMICS ON STARLIKE GRAPHS*

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Abstract. The field of epidemiology has presented fascinating and relevant questions for mathematicians, primarily concerning the spread of viruses in a community. The importance of this research has greatly increased over time as its applications have expanded to also include studies of electronic and social networks and the spread of information and ideas. The purpose of this paper is to develop techniques to analyze in detail the evolution of the systems over time. We concentrate on star graphs. We are able to obtain detailed descriptions of the dynamical behavior by a mix of convexity results and an analysis of partial fixed points arising from the corresponding differential equations. Our methods supplement other techniques in the literature by describing how the system approaches its equilibrium. Specifically, we determine the path the system takes to equilibrium as a function of the "cure" and "infection" parameters and the number of spokes n. For each n we prove the existence of a critical threshold relating the two rates. Below this threshold, the "virus" always dies out; above this threshold, all non-trivial initial conditions iterate to a unique non-trivial steady state. We end with some generalizations to other networks.
Keywords. Virus propagation, star networks, SIS model.

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