

OPTIMAL CONTROL FOR N SPECIES COMPETING SYSTEM WITH AGE-DEPENDENT AND PERIODIC VITAL RATES*

Zhi-Xue Luo and Xiao-Di Yu †‡

Abstract. In this paper, we investigate optimal policy for periodic n competing system with age-dependent. Namely, we consider the model with periodic vital rates. The existence of optimal control strategy is discussed by Mazur's theorem and optimality condition is derived by means of normal cone.

Keywords. Periodic, competing, age-dependence, optimal control, the maximum principle.

AMS Subject Classifications: 35B10, 35D05, 49J20, 65L12, 65M06.

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†Zhi-Xue Luo and Xiao-Di Yu are with Department of Mathematics, Lanzhou Jiaotong University, The People's Republic of China. E-mails: luozhixue2009@126.com.

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