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A VISCOSITY EXTRAGRADIENT METHOD FOR VARIATIONAL INEQUALITY AND SPLIT GENERALIZED EQUILIBRIUM PROBLEMS WITH A SEQUENCE OF NONEXPANSIVE MAPPINGS*

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Abstract. In this paper, we introduce a new viscosity approximation method based on the extragradient method for finding a common element of the set of solutions to a split generalized equilibrium problem, the set of fixed points of an infinite family of nonexpansive mappings and the set of solutions to the variational inequality problem for a monotone, Lipschitz continuous mapping. Moreover, the convergence theorem for sequences generated by these processes in Hilbert spaces are derived.

Keywords. Split generalized equilibrium problem, Extragradient methods, Nonexpansive mapping, Monotone mapping, Viscosity approximation method, Strong convergence

1 Introduction

The convex feasibility problem (in short, CFP), as an important optimization problem [1], is to find a common point in the intersection of finitely many convex sets. It has been applied to many areas, for instance, approximation theory [2], image reconstruction from projections [3, 4], control [5], and so on. When there are only two sets and constrains are imposed on the solutions in the domain of a linear operator as well as in this operator's ranges, the problem is said to be *the split feasibility problem* (in short, SFP) which has the following formula:

$$x^* \in C \ \text{ such that } \ Ax^* \in Q, \tag{1}$$

where C, Q are nonempty closed convex subset of Hilbert space H_1, H_2, respectively. $A : H_1 \to H_2$ is a bounded linear operator. The SFP was originally introduced by Censor and Elfving [6] for modeling inverse problems which arise from medical image reconstruction and it has also broad applications in many fields, such as signal processing, approximation theory, control theory, biomedical engineering, communications, geophysics and radiation therapy.

In the case where C and Q in (1) are the intersection of finitely many fixed point sets of nonlinear operators, problem (1) is call by Censor and Segal [7] *the split common fixed point problem* (in short, SCFP) which is a generalization of split feasibility problem and convex feasibility problem. More precisely, SCFP requires to seek an element $x^* \in H$ satisfying

$$x^* \in \bigcap_{i=1}^{p} Fix(U_i) \ \text{ such that } \ Ax^* \in \bigcap_{j=1}^{s} Fix(T_j), \tag{2}$$

where $p, s \in \mathbb{N}, Fix(U_i)$ and $Fix(T_j)$ denote the fixed point sets of two classes of nonlinear operators $U_i : H \to H, i = 1, \dots, p$ and $T_j : K \to K, j = 1, \dots, s$. Later, Moudafi [8] studied the split common fixed point problem in Hilbert spaces.

Recently, Censor et al. [9] introduced and studied some iterative methods for the following *the split variational inequality problem* (in short, SVIP): find $x^* \in C$ such that

$$\langle f(x^*), x - x^* \rangle \geq 0, \quad \forall x \in C, \tag{3}$$

and such that

$$y^* = Ax^* \in Q \ \text{ solves } \ \langle g(y^*), y - y^* \rangle \geq 0, \quad \forall y \in Q, \tag{4}$$

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